MTH 204 EXAM PREP

Note: meanings of symbols

**An element of = ⋲**

**There exist = ⋺**

**For all the values of = A (note: upside down)**

VECTOR SPACES

1. What is a Vector Space?

Ans: Let V be a non-empty set with 2 major operations an operation called Addition an an operation called Multiplication

b The V is called a vector space over the real field \_\_\_?

Ans: K

c. What are Axioms?

Ans: This checks to see if the elements satisfy the rules of being a Vector Space

e. What are the 2 types of Axioms?

Ans: Axiom of Addition

Axiom of Multiplication

f. Is 0 an element of every set/vector space?

Ans: True

g. What are the 3 Axioms of Addition?

Ans: A1: (u+v)+w = u+(v+w)

A2: (There Exist) 0 (as an element of) V - such that 0 + u = u + 0 = u

A3: (There Exist) -u (as an element of) V - such that u + (-u) = (-u) + u = 0

A4: u + v = v + u (commutativity)

h. What are the 6 Axioms of Multiplication?

Ans: A1: k\*(u + v) = ku + kv for any scaler k (is an element of) K

A2: (k₁+k₂)\*w = k₁w +k₂w for any scaler k₁,k₂ (is an element of) K

A3: (k₁k₂)\*v = k₁\*(k₂v) for any scaler k₂ (is an element of) K

A4: 1u = u for the unit scaler 1 (is an element of) K

A5: (u \* v)\*k₂ = u\*(v\*k₂)

A6: u¯¹ \* u = 1 (There Exist) u¯¹ (as an element of) V

i. The first form of the Axioms are concerned with the \_\_\_\_\_\_\_\_\_\_\_\_?

Ans: Additive Structure of V

j. The second form of the Axioms are concerned with the \_\_\_\_\_\_\_\_\_\_\_\_\_?

Ans: Action of field K or scalars on the vector space V

EXAMPLES OF VECTOR SPACES

2. What are the 4 examples of Vector Spaces?

Ans: Space Kⁿ

Matrix Space Mⁿ,ⁿ (1st n is actually an m)

Polynomial Space P(t)

Function Space F(x)

A. Space Kⁿ

b. Write a line or two about Space Kⁿ?

Ans: Let K be an arbitrary field.

The notation Kⁿ is used to denote the set of elements of all n-tuples in K (eg: K = {Kⁿ}; Kⁿ = u, v, w, e.t.c)

Here Kⁿ is a vector space over K

b1. Show an example to prove that a Space Kⁿ is a vector space?

Ans: vector addition

= (a₁+a₂+a₃), (b₁+b₂+b₃)

= (a₁+b₁, a₂+b₂ a₃+a₃) Addition Proved✅

scalar multiplication

= k(a₁,a₂, ... an)

= (ka₁, ka₂, ... kan)

= 0(a₁,a₂, ... an)

= (0, 0, ..., 0)

= -(a₁,a₂, ... an)

= (-a₁,-a₂, ... -an) Multiplication Proved✅

B. Polynomial Space

c. Write a line or two about Polynomial P(t)?

Ans: Let P(t) denote all the sets of polynomials in form.

P(t) = (a₀ + a₁ + a₂t² + a₃t³)

c1. Show an example to Prove that a Polynomial Space P(t) is a vector space?

Ans: vector addition

p(t) = (a₀ + a₁t + a₂t² + a₃t³), q(t) = (b₀ + b₁t + b₂t² + b₃t³)

= p(t) + q(t) = [(a₀+b₀), (a₁+b₁)t + (a₂+b₂)t² + (a₃)t³] Addition Proved✅

scalar multiplication

p(t) = (a₀ + a₁t + a₂t² + , ..., + a₃t³)

K\*p(t)

= k(a₀ + a₁t + a₂t² + ... + a₃t³)

= (ka₀ + ka₁t + ka₂t² + ... + ka₃t³) Multiplication Proved✅

C. Matrix Space

d. Write a line or two about Matrix Space Mⁿ,ⁿ?

Ans: The notation Mⁿ,ⁿ or M is used to denote the set of all m\*n matrices with entries in the field K

d1. Show an example to Prove that a Matrix Space Mⁿ,ⁿ is a vector space?

Ans: vector addition

a₁₁ a₁₂ + b₁₁ b₁₂

a₁₂ a₂₂ + b₁₂ b₂₂

= a₁₁+b₁₁ a₁₂+b₁₂

a₁₂+b₁₂ a₂₂+b₂₂ Addition Proved✅

scalar multiplication

= k(a₁₁ a₁₂)

(a₁₂ a₂₂)

= (ka₁₁ ka₁₂)

(ka₁₂ ka₂₂) Multiplication Proved✅

D. Function Space

e. Write a line or two about Function Space F(x)?

Ans: Let x be a non-empty set and let F be an arbitrary value. Then let F(x) denote the set of all function x into k

Then F(x) is a vector space over K

e1. Show an example to Prove that a Matrix Space Mⁿ,ⁿ is a vector space?

Ans: vector addition

Let f & g be functions in F(x)

= (f + g) (x)

= f(x) + g(x) (for all the values of) x (is an element of) X Addition Proved✅

scalar multiplication

= (k⨍)(x) = k⨍(x) (for all the values of) x (is an element of) X Multiplication Proved✅

SUBSPACE

f. Define a Subspace?

Ans: Let V be a vector space over the field K, and W be a subspace V if W itself is a vector space with respect to the operations of

vector addition and scalar multiplication

f1. State the Theorem 1?

Ans: Suppose W is a subset of a vector space V. Then W is a subspace of V if the following two conditions hold

1. The Zero vector 0 belongs to W

2. (for all the values of) u, v, w (are members of W) | K (is a member of) K

a) The sum of u+v (is a member of) W

b) The multiple of ku (is a member of) W

LINEAR DEPENDENCE AND INDEPENDENCE

g. Define Linear Dependence & Linear Independence? (Answers in note)

Ans: We say that the vectors v1, v2, ... vm in V are Linearly Dependent if there exist scalars a1, a2, ..., am in K, not all of them

such that our a1v1, + a2v2, ..., + amvm = 0

otherwise we say that the vectors are linearly Independent

g1. Solve the following vectors and show that they are linearly dependent or independent?

i. x1 = (1,1,1,3), x2=(-1,0,-5,-6), x3=(1,2,-1,2), x4=(-1,0,2,1), x5=(1,1,-1,1)

ii. (w,1,0),(0,w,0), & (0,0,w+3)

g2. Show that the following vectors generate R³

i. v1=(1,1,1), v2=(0,1,1), v3=(0,1,-1) (first step: unlike linear -> do a scalar multiplication)

BASIS

f. Define Basis? (Answers in note)

Ans: Basis of a vector space is a linearly independent subset which generates or spans the whole space

f1. What are the two (2) conditions to basis?

i) The set generates or spans V

ii) The set is linearly independent

f2. Show that in a vecror Space Rⁿ. The following set of vectors is a basis for Rⁿ (Answers in note)

i) e1 = (1,0,0,...,0), e2=(0,1,0,...,0), e=(0,0,...,1)